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Numerical Solution for the Deformations of Tunnels in Rock Masses and its Application to Estimate In Situ Stress Ratio K_0

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In this paper, the finite element method is used to obtain deformations of a rock tunnel. More than 3,000 numerical simulations and calculations were carried out. The analysis of this results has allowed to achieve mathematical formulations and graphs, to obtain in advance support monitored displacements in tunnelling construction, so they can be predicted before excavation and supporting woks. The application of the presented formulations and charts will aid engineers to control tunnel monitoring, reducing decision time when problems occur.
The analysis is limited to cases involving elastic behaviour of the terrain for tunnels of average to good quality in rock, and to excavation sections of average or smaller dimensions for road tunnels.
The results show that, in all cases, mathematical expressions to obtain induced displacements around tunnel excavations, have similar pattern than Kirsch (1898) expressions for circular excavations.
As corollary of the achieved results, a method to estimate in situ natural geostress ratio $K_0=\sigma_h/\sigma_v$, by means of convergence measurements, is presented.

In diesem Beitrag wurden die Verschiebungen eines Tunnels durch die Methode de Finiten Elemente berechnet. Mehr als 3000 numerische Simulationen und Berechnungen wurden ausgeführt.
Die Analyse dieser Resultate hat es ermöglicht, mathematische Formeln und Grafiken zu entwickeln, die zur Voraussage der Verschiebungen in Tunnel dienen, auch unter Betrachtung der verschiedenartigen Abstützungen. So können die Verschiebungen vor den Erdarbeiten und dem Bau der Abstützung abgeschätzt werden. Die abgeleiteten Formeln und Grafiken werden vor allem einen grossen Dienst für Ingenieure bei der Mithilfe zu zeitsparenden Entscheidungen im Tunnelbau und bei der Tunnelkontrolle leisten.
Die analysierten Fälle sind auf eine elastische Modellierung des Baugrunds beschränkt, sowie auf eine mittlere bis gute Qualität des Felsens und auf die Aushebung von Abtragungsabschnitten mittlerer oder geringer Grösse für den Strassenverkehr.
Die Resultate zeigen, dass in allen Fällen die hergeleiteten Formeln zur Berechnung der durch den Tunnelbau verursachten Verschiebungen um den Tunnel die gleiche mathematische Struktur haben wie diejenigen von Kirsch (1898) für kreisförmigen Bodenaushub.
Als Korollarium der ausgeführten Berechnungen wird eine Methode zur Abschätzung der vorherrschenden in situ Geospannungs-Verhälniss $K_0=\sigma_h/\sigma_v$ angegeben, die von den Messungen der Verschiebungen im Tunnel ausgeht.

Dans cet article, la méthode des éléments finis est utilisée pour obtenir les déformations d'un tunnel creusé dans la roche. Plus de 3000 simulations numériques et calculs ont été effectuées. L'analyse de ces résultats a permis de définir des modèles mathématiques et des abaques afin de prédire, avant l'excavation et les travaux de soutènement, les déplacements auscultés pendant la construction du tunnel. L'application de ces modèles et abaques permettra donc aux ingénieurs de contrôler les déplacements durant l'auscultation du tunnel et de réduire le temps de décision lorsque des problèmes surviennent.
L'analyse est limitée au cas du comportement élastique du terrain pour des tunnels creusés dans de la roche de moyenne a bonne qualité, et à de sections d'excavation de petites à moyennes dimensions pour des tunnels routiers.
Les résultats montrent que, dans tous les cas, les expressions mathématiques permettant d'obtenir les déplacements induits autour des excavations de tunnel, présentent des similitudes avec celles de Kirsch (1898) pour des excavations circulaires.
Comme corollaire aux résultats obtenus, une méthode pour estimer le rapport des contraintes géostatiques in situ $K_0=\sigma_h/\sigma_v$ au moyen de mesures de la convergence, est exposée.

Introduction

In determining the perimeter movements of a tunnel, numerical methods simulating the stresses and deformations around the excavation are often used. Typically, a finite element model is used to handle the large volume of data and generated results.

The analysis by finite element models of tunnels in rock having a moderate overburden is a well-defined process, especially for tunnels driven in homogeneous terrain. For such conditions, this paper established some numerical expressions that directly provide perimeter movements of tunnels for the most commonly used tunnel sections. The methodology used to determine the obtained expressions required the performance of many calculations in order to consider the cases that are most frequently encountered in tunnel design and construction. It has also been possible to verify that these expressions are similar to existing expressions obtained analytically for the case of a circular tunnel in a rock mass with homogeneous stress.

The obtained expressions can be used to easily determine the expected deformations in a given tunnel within the scope of cases noted above, i.e., circular tunnels in homogeneous rock, with moderate overburden. Using these expressions can eliminate the need to perform complicated numerical analyses to determine tunnel deformations. Instead, when the back-analysis interactive process is carried out, the parameters of the rock can be deduces starting from the results of the tunnel measurements, and using the expressions and graphics presented in this paper.

Another interesting contribution of the work carried out is the ability to obtain a simple ratio between the coefficient of the horizontal and diagonal convergences measured in a tunnel and the value of the rock coefficient K_0, that is, the relation between horizontal stresses at a point in the rock mass.This parameter is difficult to measure directly.

Analytical Solution

The differential equations of elasticity, applied to the case of tunnel excavation in an elastic medium, can be solved analytically only in certain carefully defined cases. The most simple case is the one corresponding to a circular tunnel in an infinite medium and loaded with an initial stress σ^0 equal in all directions, that is, with coefficient $K_0=1$ and without considering the increase in stresses with the depth due to the terrain weight. The solution to this problem was determined by Kirsh (1898), and is the one expressed in the following way in reference to the relative movement in the radial direction to the tunnel perimeter area:

$$u_R = [(1+\nu)/E]*\sigma^0*R \qquad (1)$$

where:

u_R = Radial movement of the perimeter area of the tunnel
E = Rock mass deformation modulus
ν= Rock mass Poisson's ratio
σ^0 = Initial stress of the rock
R = Tunnel radius

We can simply deduce from expression (1) that the radial movement in a tunnel, or the convergence, is directly proportional to the tunnel radius and the initial stress status and inversely to the rock mass deformation modulus. In other words, the convergences of a tunnel are greater for larger tunnels at greater depths, and lesser with a poorer quality of rock.

Subsequently, these results were applied in general for the case of non-isotropic initial stress (Kirsch, 1898) and for an elliptical-shaped tunnel (Inglis, 1913). An analytical solution in displacements has been developed for the circular tunnel with any K_0 obtained by Pender (1980).

For the case of radial movement in the perimeter area of the tunnel, the expression is:

$$u_R = [(1+\nu)/E]*\sigma^0*R*[1/2(1+ K_0)-3/2(1-K_0)(1-\nu)*\cos 2\theta] \qquad (2)$$

where θ shows the point at which the movement is measured, and K_0 is the existing relation between the horizontal and vertical stresses on each point. Making $\theta=0$ and $\theta=\pi/2$ in (2) gives, respectively, the horizontal convergence and roof settlement of the tunnel. If we call the radial movement obtained for the case of $K_0 = 1$ obtained from (1) basic movement, the following expressions remain for the roof settlement and the horizontal convergence:

$$R_S = u_0*[1/2(1+ K_0)+3/2(1-K_0)(1-\nu)] \qquad (3)$$

$$Ch = u_0*[(1+ K_0)-3(1-K_0)(1-\nu)] \qquad (4)$$

where u_0 is obtained from:

$$u_0 = [(1+\nu)/E]*\sigma^0*R \qquad (5)$$

Note that in order to obtain the horizontal convergence Ch in expression (4), the value which is directly obtained from (2) has been multiplied by 2, since the measured value in-situ is actually the relative movement between both sides of the tunnel, so by symmetry it is twice the displacement of each one of them.

In both cases, the movements are expressed as the product of the basic movement u_0 multiplied by an anisotropy coefficient, the latter which appears due to the non-isotropic distribution of the initial stresses around the peripheral area of the tunnel. We will represent this anisotropy coefficient with the Greek letter kappa κ, which is a function of K_0, ν and the considered movement.

Designating u_i to any of the movements that we may consider in the perimeter area of the circular tunnel, whether by the displacement of a point, or by the relative movement between two of them, we can express in general, that:

$$u_i = u_0 * \kappa_i(K_0, \nu) \qquad (6)$$

where κ_i is the corresponding anisotropy coefficient to the considered movement.

In the case of the horizontal convergence and roof settlement (expressions (3) and (4)), in Figure 1, the value of κ has been represented as a function of ν and K_0. In both cases a set of lines appears, and then it passes by the point of unit ordinate when $K_0=1$. That is, when $K_0=1$ there is no influence of ν on the value of κ, and the movement produced is that which has been denominated basic u_0.

We should also note that there is a certain value for K_0 which cancels out the roof settlement, and another value for K_0 which cancels out the tunnel's horizontal convergence for a given ν coefficient. From (3) and (4) we can obtain these two values for K_0 according to the following expressions:

$$K_0(R_S = 0) = (3\nu\text{-}4)/(3\nu\text{-}2) \qquad (7)$$

$$K_0(Ch = 0) = (3\nu\text{-}2)/(3\nu\text{-}4) \qquad (8)$$

Note that they are inverse values, that is, if we call the value for K_0 which cancels the tunnel roof settlement K_0 critical or K^{CR}, then its inverse $1/K^{CR}$ cancels out the horizontal convergences of such a tunnel.

Numerical Solution

Expression (6) is generally within the cases involving elastic behavior of the terrain for tunnels considered up to now, and can be analytically obtained for the simplified case of a circular tunnel without considering the weight of the terrain, that is, with uniform initial stresses throughout the rock mass.

By means of the finite element analysis described below, a generalization of (6) has been carried out for cases nearer to reality, that is, tunnels of shapes not necessarily circular, taking into consideration the weight of the terrain and the variation of the initial stress status with the depth. Using different models of finite elements, various numerical calculations have been made. It has been verified that the expression that is obtained for the movements of the perimeter area of the tunnel is formally the same as expression (6).

The analysis was carried out through the resolution of 7200 different models of finite elements, using the ANSYS commercial software version 5.2. Figure 2 shows one of the models of elements used. With these models, we intended to include the most common range of variation of each of the parameters that pertain to the problem. The parameters which have been considered as variables are described below, and are summarized in Table I.

Table I: Variation range of parameters

PARAMETER	RANGE
Excavation section	20 - 60 m^2
Elastic Modulus	5 – 50 GPa
Poisson's ratio	0.15 - 0.35
Coefficient K_0	0.5 - 2.5
Mass density	2400 - 2600 Kg/m^3
Overburrden	50 - 250 m
Tunnel shape	Circular, horseshoe and broad

Tunnel Shape and Dimensions

Four different tunnel shapes have been calculated, which are the most common in roads, railroads or hydraulic works: circular tunnel, horseshoe-shaped tunnel and semicircular tunnel. A different set of expressions has been obtained for each one of these four shapes, as described in section 4.

The tunnel dimensions have been varied by means of the excavation surface S, instead of by the radius R. In this way the non-circular shapes have been more appropriately considered. The excavation section has been varied between 20 and 60 m^2, as more representative limits in half-section excavations or first stage sections. In any case, the obtained results are applicable to greater sections if they are maintained within the elastic range.

Depth and Stress Status

The models of finite elements which were used comprise an area of approximately three diameters around the excavation. In all the cases the terrain surface remains out of the model, that is, shallow tunnels have not been considered. We have considered depth values between 50 and 250 meters over the center of gravity of the section.

In the model that we have used, the initial stresses prior to the excavation were determined by the following expressions:

$$\sigma_V = \rho * g * h \qquad (9)$$

$$\sigma_H = K_0 * \sigma_V \qquad (10)$$

where σ_V and σ_H are the vertical and horizontal stresses, respectively, ρ is the terrain mass density, g the gravitational acceleration, h the depth at each point, and K_0 the ratio between horizontal and vertical stresses. The expression for the vertical stress is the hypothesis most commonly used, and has been cited by Herget (1988), Hoek & Brown (1980) and Wittke (1990) among others.

The calculation input parameters are the density, which has been considered between 2400 and 2600 Kg/m^3, and the K_0 coefficient, for which the values between 0.5 and 2.5 have been used.

Terrain Deformability

The most limited parameter is the terrain Poisson's ratio, ν, which has been varied between 0.15 and 0.35.

In reference to the elastic modulus E, we have chosen to use values between 5 and 50 GPa. This range excludes poor quality rocks, which would require a different treatment because in poor quality rock some plastification may appear around the tunnel as a consequence of the excavation.

In all of these cases, a purely elastic behavior of the rock has been assumed, which is nearly approximate to the case of tunnels of average or small dimensions and in rock mediums of average to good quality.

Obtained Expressions

Movement Determination

Within all the possible movements that can be measured in the perimetric area of a tunnel, the movements for which a determining expression is given are the following: the roof settlement, the horizontal convergence and the diagonal convergence. For all of them, the expression obtained is the following:

$$u_i = u_0 * \kappa_i(K_0, \nu, shape) \qquad (12)$$

with:

$$u_0 = [(1+\nu)/E] * \gamma * H * \sqrt{(S/\pi)} \qquad (13)$$

where:

u_i: any of the three mentioned movements
κ_i: the anisotropy coefficient, which depends on the considered movement, the K_0 coefficient, Poisson's ratio and the tunnel shape
u_0: the basic movement, as expressed in (13), where each parameter represents:
E: Rock mass elastic modulus
ν: Rock mass Poisson's ratio
γ: Rock mass specific weight, $\gamma=\rho*g$
H: Depth of the gravity center of the tunnel section
S: Cross-section area of the tunnel excavation

Figures 1, 2, and 3 allow the graphic determination of κ for the different tunnel shapes which have been studied.

These graphics reflect the important influence that the value of K_0 has on the tunnel deformations. When K_0 increases, the roof settlements tend to decrease and the horizontal convergences increase, while the diagonal convergences are produced as a composition of the other two movements.

The scope of validity of these numerical determinations is limited by the starting suppositions used in the numerical models. In general, we can state that expression (12) is valid when the behavior of the terrain is maintained in the elastic range and the rock mass is sufficiently homogeneous. The following scope of validity should be noted:

- Tunnel of any of the four shapes studied or other similar ones
- Small or average excavation sections (<80-100 m2), so that the behavior of the mass could be assumed as elastic
- Moderate depth, so that there is no surface proximity influence on one hand, and no plastification phenomena or rock swelling on the other hand
- Average or good quality rock, given from the beginning an RMR higher than 40, so that its behavior could be considered elastic
- Terrain with approximately homogeneous behavior in all directions.

Consideration of the face distance

The generic expression (12) provides the total movement that has occurred, starting from the initial status, prior to the tunnel excavation, until the conclusion of it. Nevertheless, if we take into account that part of the movement is produced before making the first reading of the convergences, the movement measured is never the full quantity, but merely a certain fraction of it. This fraction depends on the face quality and on the face distance of the first reading, and can be obtained as the product of the total movement multiplied by the parameter (1-λ), as defined by Panet & Guenot (1982).

In such a case, the expression of the measured movements will remain as follows:

$$u_i = u_0 * \kappa_i * (1-\lambda) \qquad (14)$$

where parameter (1-λ) is obtained from Figure 7, as a function of the distance to the face from the convergences measurement section when it was first measured. Of all the curves mentioned by Panet & Guenot, in this case the one corresponding to Ns = 1 should be used, which is the one for good quality terrain with no apparent plastification.

Comparison with the analytical expression

In the case of the circular tunnel, the results derived from expression (12) and Figure 3 can be easily compared to the analytical expressions (3) and (4). The following differences and analogies can be pointed out:

- In both cases the value of κ depends on K_0 and on ν, and in both cases they have the same influence: with a greater K_0, lesser roof settlements and greater horizontal convergences; and with a greater ν, lesser influence of K_0.

- In both the analytical and the numerical determinations, there is a certain K0 in which there is no influence of ν in the value of the anisotropy coefficient κ, that is, the represented straight lines always pass by the same point. In the circular tunnel, this point corresponds to $K_0 = 1$.
- The values obtained by the application of expression (12) for the roof settlement give movements somewhat greater than the ones obtained in expression (3). This result may be due to the model used and to the different distribution of initial stress.
- Figure 3 shows that there is a critical value for K_0 which cancels out the roof settlement, and that the inverse of such a value cancels out the horizontal convergence. This circumstance is also deduced from the analytical expressions (7) and (8), although the values for K^{CR} are slightly different.

Ratio between two convergence measurements

In expression (14) we can calculate the existing relation between any two different tunnel movements. As u_0 and λ are equal for the movements measured at the same moment, we have the following:

$$u_i/u_j = \kappa_i/\kappa_j \qquad (15)$$

Therefore, we can state that the relation existing between two tunnel movements measured at any time is equivalent to the ratio of the corresponding aniostropy coefficients.

This result is of great interest for obtaining the value of the parameter K_0 for the rock mass starting from the tunnel monitoring measurements. In fact, we can estimate the ratio of the anisotropy coefficients for the cases of horizontal and diagonal convergences, and to represent it as a function of K_0 and ν.

In Figures 4, 5 and 6, graphics are shown for each type of excavation analyzed. We can conclude that measuring the horizontal and diagonal convergences in a tunnel, and calculating the relation between them, these graphics allow us to determine the coefficient K_0 of the rock mass for a given value of ν.

Model errors

In order to estimate the errors inherent in the model used, derived from the problematic numerical resolution itself, in some cases a more exact model has been used, with the model boundary most distant from the excavation, and with a much more dense discretization.

The results of these more exact calculations, compared with the ones from the initial model, show that the horizontal and diagonal convergences are practically the same, while there are changes in the roof settlements. The error increases as the depth increases, reaching a maximum of a 15% error, and obtaining lesser movements in the more exact calculations.

Consequently, the roof settlements that are shown in this paper could be greater than the real values, but in any case, the difference would not be greater than 15%.

Conclusions

The results have been shown in graphic form in which we note the remarkable influence of coefficient K_0 of the rock mass and of Poisson's ratio ν, for the determination of the anisotropy coefficient κ as well as for the determination of deformations in the rock of the perimetric area. Through the ratio between horizontal and diagonal convergences measured during the construccion of a tunnel, we can determine the value of parameter K_0 of the rock mass.

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Appendix 1 (Graphics and charts)

FIGURE 1: κ (CIRCULAR TUNNEL)

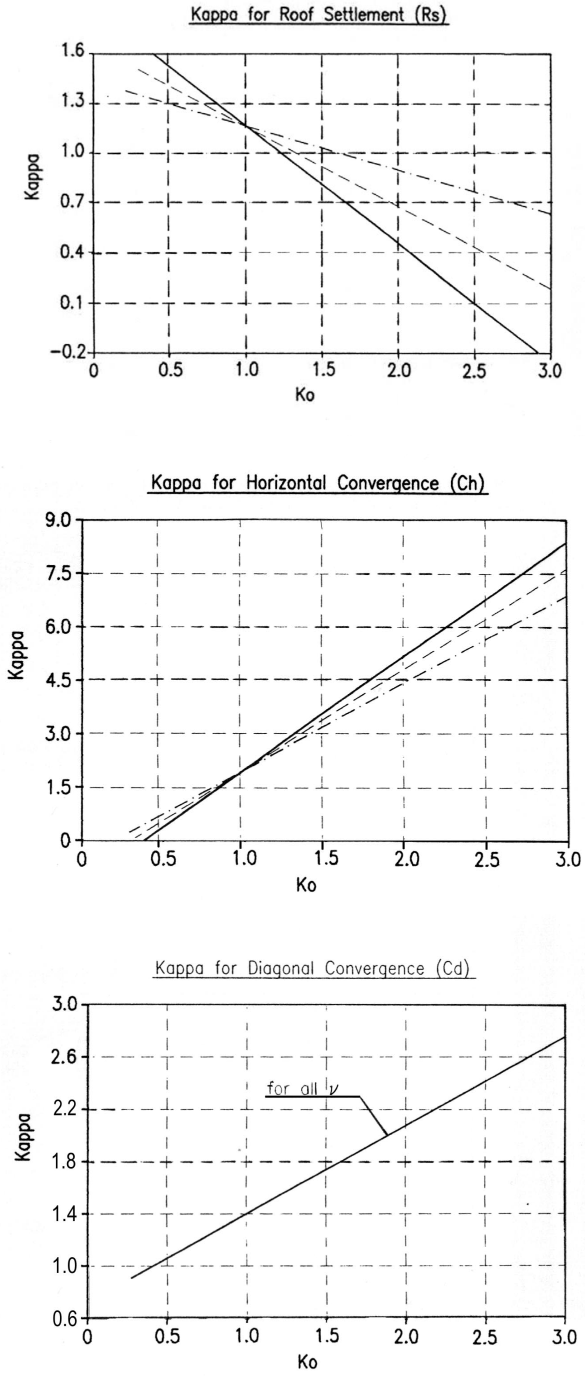

FIGURE 2: κ (HORSE-SHAPE TUNNEL)

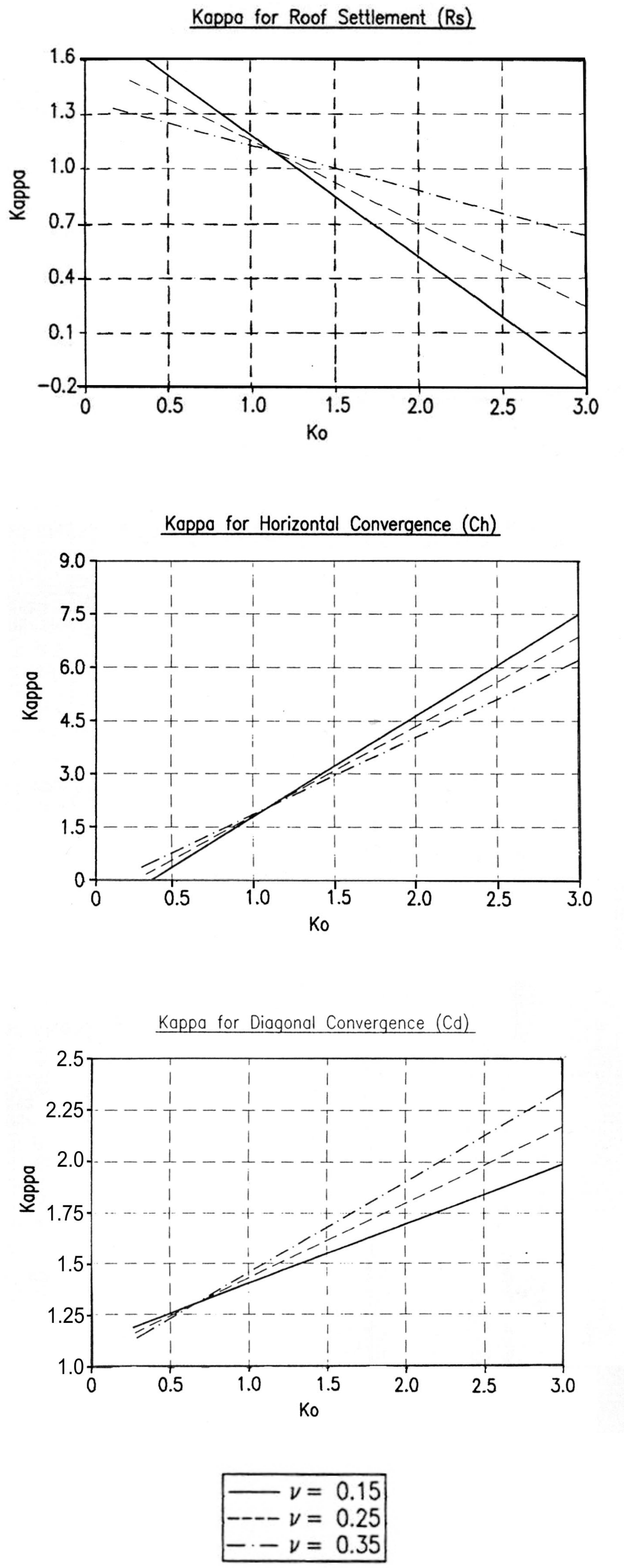

FIGURE 3: κ (BROAD TUNNEL)

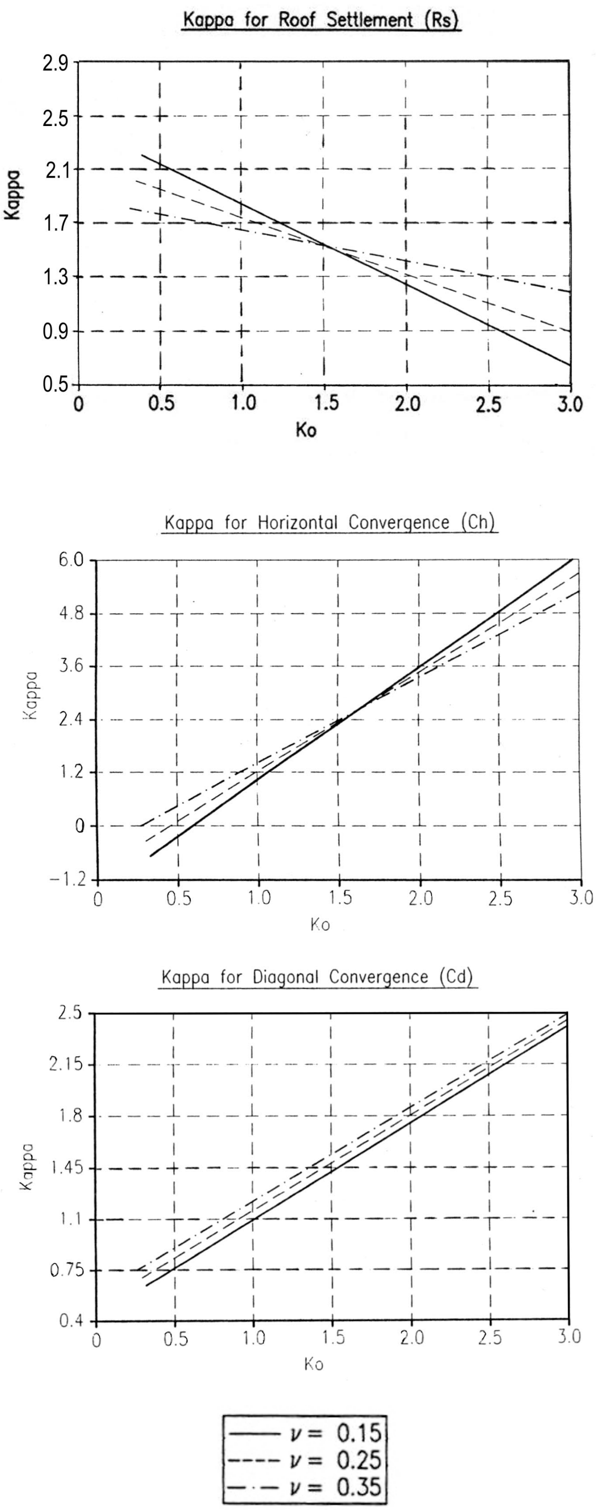

FIGURE 4: K_0 (CIRCULAR TUNNEL)

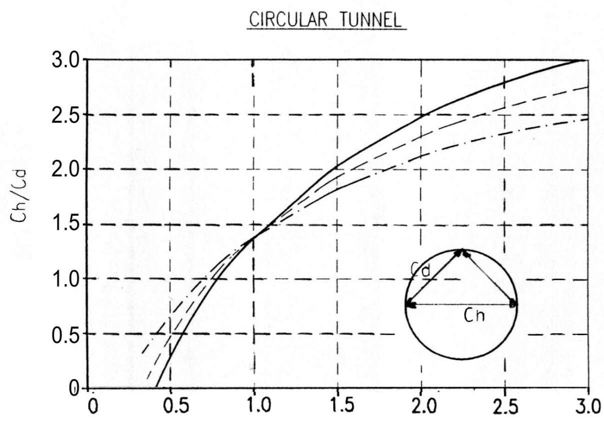

FIGURE 5: K_0 (HORSE-SHAPE TUNNEL)

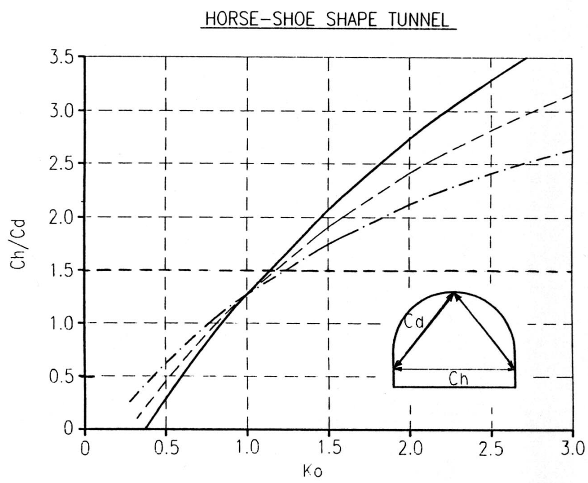

FIGURE 6: K_0 (BROAD TUNNEL)