Rock Engineering and Rock Mechanics: Structures in and on Rock Masses – Alejano, Perucho, Olalla & Jiménez (Eds)

Procedure for obtaining and analyzing the diametric deformation of a tunnel by means of tape extensometer convergence measures

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ABSTRACT: The diametric deformation of a tunnel (the ratio of tunnel wall displacement vs. tunnel radius) is a very useful parameter to analyze its tenso-deformational behavior. In this paper we present a new methodology for use it as monitoring variable to assess tunnel stability. It has obtained an analytical expression useful to obtain the diametric deformation from tape extensometer convergence measures. A procedure to analyze the diametric deformation is proposed. A practical example is carried out.

1 INTRODUCTION

Typically, the measurement of convergences is the principal monitoring method for tunnels excavated in rock. The convergence is measured with a tape extensometer, and it is a straightforward, simple, fast and cheap method that it does not require the installation of sophisticated instrumental devices or indirect measurement. Furthermore, the measuring interferes very little in the construction of the tunnel. All these advantages allow the installation of a large number of benchmarks along the tunnel, which is an additional advantage, because the heterogeneous behavior that usually shows a rock mass.

However, like the rest of tunnel monitoring methods is always difficult to perform an analysis of the measurements obtained, especially if the number of data is large. The tunnel geomechanical monitoring should always tries to evaluate whether the measures indicate the existence of instabilities, if it is required the reinforcement of the support or whether they indicate the stabilization of the tunnel.

Usually convergence analysis provides a good evaluation of the degree of stabilization that a tunnel has reached. On the other hand, does not always provide a good indication of the state of charge of support or the existence of deformational instabilities. Often it is not easily transpose the results obtained by numerical methods to measure movements; these movements will depend on where the convergence's rings are installed.

In this paper, it is presented a methodology for monitoring of tunnels that is the use of Diametric Deformation (ε_ϕ) as a monitoring variable. For this purpose, first it is described the calculation of the Diametric Deformation based on convergence measures. Here is suggested a method of analysis of Diametric Deformations in order to obtain the alarm levels for monitoring the execution of the tunnel and an estimation of the load level at which the support is subjected. Finally we present a case of a tunnel, where the proposed methodology is used

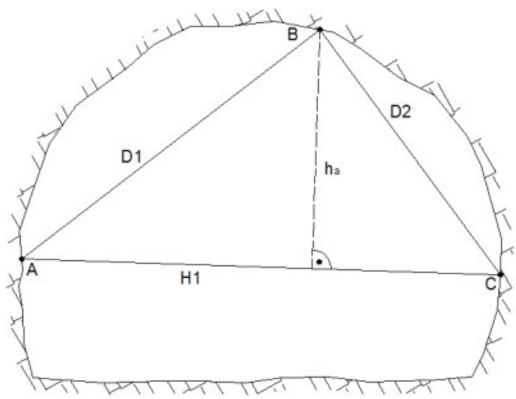

Figure 1. Benchmarks section.

2 CONSTRUCTION OF A CIRCUMFERENCE OF REFERENCE

ABC is a triangle whose vertices represent the benchmarks used for monitoring convergence, see Figure 1.

Following the nomenclature commonly used, the lines that are measured with a tape convergence would be designated as follows:

$$\left.\begin{aligned} H1 &= AC \\ D1 &= AB \\ D2 &= BC \end{aligned}\right\} \tag{1}$$

h_a is the height of the triangle ABC, built on the H1 side. This height is perpendicular to the side of the

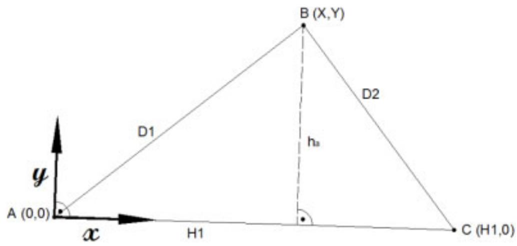

Figure 2. Coordinate system.

triangle which is constructed and passes through the vertex B. For any triangle, it is verified:

$$h_a = \frac{\tau}{H1} \tag{2}$$

where

$$\tau = \frac{1}{2}\sqrt{(D2+H1-D1)(D2-H1+D1)(-D2+H1+D1)(D2+H1+D2)} \tag{3}$$

In order to obtain the analytical equation of the circle circumscribing the triangle ABC, we define a coordinate system, see Figure 2, with origin at the vertex A, x-axis in the direction defined by the segment AC and y-axis perpendicular to the previous and therefore parallel to the height h_a.

In this reference system, the coordinates of the vertices of the triangle of convergences are:

$$\left.\begin{array}{l} A \equiv (x_1, y_1) = (0,0) \\ B \equiv (x_2, y_2) = (X, Y) \\ C \equiv (x_3, y_3) = (H1, 0) \end{array}\right\} \tag{4}$$

where the coordinates of the vertex B, are easily obtained by the Pythagorean Theorem:

$$\left.\begin{array}{l} X = \sqrt{D1^2 - h_a^2} = \sqrt{D1^2 - \dfrac{\tau^2}{H1^2}} \\ Y = h_a = \dfrac{\tau}{H1} \end{array}\right\} \tag{5}$$

The equation of a circle passing through three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is defined algebraically by the following equation:

$$\begin{vmatrix} x & y & x^2+y^2 & 1 \\ x_1 & y_1 & x_1^2+y_1^2 & 1 \\ x_2 & y_2 & x_2^2+y_2^2 & 1 \\ x_3 & y_3 & x_3^2+y_3^2 & 1 \end{vmatrix} = 0 \tag{6}$$

Developing this determinant:

$$M_1 x - M_2 y + M_3 (x^2 + y^2) - M_4 = 0 \tag{7}$$

where:

$$M_1 = \begin{vmatrix} y_1 & x_1^2+y_1^2 & 1 \\ y_2 & x_2^2+y_2^2 & 1 \\ y_3 & x_3^2+y_3^2 & 1 \end{vmatrix} \tag{8}$$

$$M_2 = \begin{vmatrix} x_1 & x_1^2+y_1^2 & 1 \\ x_2 & x_2^2+y_2^2 & 1 \\ x_3 & x_3^2+y_3^2 & 1 \end{vmatrix} \tag{9}$$

$$M_3 = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \tag{10}$$

$$M_4 = \begin{vmatrix} x_1 & y_1 & x_1^2+y_1^2 \\ x_2 & y_2 & x_2^2+y_2^2 \\ x_3 & y_3 & x_3^2+y_3^2 \end{vmatrix} \tag{11}$$

Dividing in Equation 7 by M_3 and rearranging:

$$x^2 + y^2 + \frac{M_1}{M_3} x - \frac{M_2}{M_3} y - \frac{M_4}{M_3} = 0 \tag{12}$$

or

$$x^2 + y^2 + D\,x + E\,y + F = 0 \tag{13}$$

where:

$$\left.\begin{array}{l} D = \dfrac{M_1}{M_3} \\ E = \dfrac{-M_2}{M_3} \\ F = \dfrac{-M_4}{M_3} \end{array}\right\} \tag{14}$$

Developing the determinants, we obtain:

$$\begin{aligned} M_1 = {} & y_1 (x_2^2 + y_2^2) + y_3 (x_1^2 + y_1^2) + y_2 (x_3^2 + y_3^2) - \\ & - y_3 (x_2^2 + y_2^2) - y_2 (x_1^2 + y_1^2) - y_1 (x_3^2 + y_3^2) \end{aligned} \tag{15}$$

$$\begin{aligned} M_2 = {} & x_1 (x_2^2 + y_2^2) + x_3 (x_1^2 + y_1^2) + x_2 (x_3^2 + y_3^2) - \\ & - x_3 (x_2^2 + y_2^2) - x_2 (x_1^2 + y_1^2) - x_1 (x_3^2 + y_3^2) \end{aligned} \tag{16}$$

$$M_3 = x_1 y_2 + x_3 y_1 + x_2 y_3 - x_3 y_2 - x_2 y_1 - x_1 y_3 \tag{17}$$

$$\begin{aligned} M_4 = {} & x_1 y_2 (x_3^2 + y_3^2) + x_3 y_1 (x_2^2 + y_2^2) + x_2 y_3 (x_1^2 + y_1^2) - \\ & - x_3 y_2 (x_1^2 + y_1^2) - x_2 y_1 (x_3^2 + y_3^2) - x_1 y_3 (x_2^2 + y_2^2) \end{aligned} \tag{18}$$

Substituting for the values of the coordinates of each vertex, see Equations 4 and 5, is simplified significantly:

$$M_1 = Y H1^2 = \tau H1 \tag{19}$$

$$\begin{aligned} M_2 &= X H1^2 - H_1 (X^2 + Y^2) = \\ &= H1 \left[\sqrt{D1^2 H1^2 - \tau^2} - D1^2 \right] \end{aligned} \tag{20}$$

$$M_3 = -Y\,H1 = -\tau \tag{21}$$

$$M_4 = 0 \tag{22}$$

Therefore the parameters of Equation 13 would remain:

$$\left.\begin{aligned} D &= \frac{M_1}{M_3} = -H1 \\ E &= \frac{M_2}{M_3} = \frac{H1}{\tau}\left[\sqrt{D1^2 H1^2 - \tau^2} - D1^2\right] \\ F &= 0 \end{aligned}\right\} \tag{23}$$

On the other hand, the equation of a circle with center (x_0, y_0) and radius R, is given by the expression:

$$(x - x_0)^2 + (y - y_0)^2 = R^2 \tag{24}$$

Developing and rearranging:

$$x^2 + y^2 - 2x_0\, x - 2y_0\, y + x_0^2 + y_0^2 - R^2 = 0 \tag{25}$$

Identifying terms in Equation 13:

$$\left.\begin{aligned} x_0 &= -\frac{D}{2} \\ y_0 &= -\frac{E}{2} \\ R &= \sqrt{x_0^2 + y_0^2 - F} \end{aligned}\right\} \tag{26}$$

Substituting the values obtained in Equation 23:

$$\left.\begin{aligned} x_0 &= \frac{H1}{2} \\ y_0 &= -\frac{H1}{2}\left[\sqrt{D1^2\, H1^2 - \tau^2} - D1^2\right] \\ R &= \sqrt{x_0^2 + y_0^2} \end{aligned}\right\} \tag{27}$$

Operating into this equation:

$$\begin{aligned} R^2 &= \frac{H1^2}{4} + \frac{H1^2}{4\tau^2}\left[\sqrt{D1^2\, H1^2 - \tau^2} - D1^2\right] = \\ &= \frac{H1^2\, D1^2}{4\tau^2}\left[H1^2 + D1^2 - 2\sqrt{D1^2 H1^2 - \tau^2}\right] \end{aligned} \tag{28}$$

Therefore, the diameter of the circumference defined by three lengths of the benchmarks section is given by the expression:

$$\phi = \frac{H1\, D1}{\tau}\sqrt{H1^2 + D1^2 - 2\sqrt{D1^2 H1^2 - \tau^2}} \tag{29}$$

3 DIAMETRIC DEFORMATION

Once three points have been materialized in a tunnel section, by means of the benchmarks, it is possible to determine a reference circumference whose diameter is given by Equation 29.

According to the tunnel face progress away from the monitoring section; this will deform the reference circumference. As the three measuring points are fixed, it is possible to calculate the new diameter of the circumference deformed, as illustrated in Figure 3.

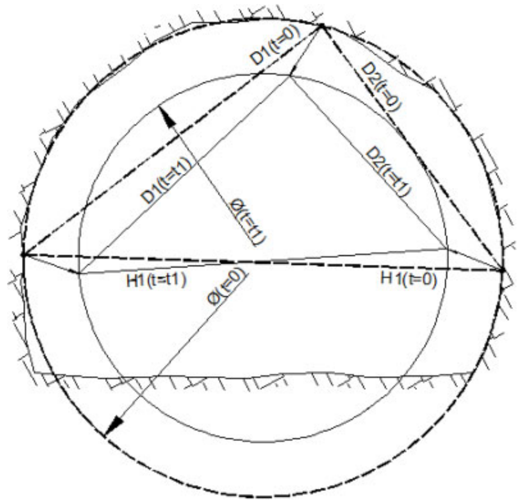

Figure 3. Deformation of the convergence section.

Therefore is immediately obtaining the deformation suffered in the rock mass, simply by comparing the diameter deformed of the circumference with the initial.

$$\varepsilon_\phi(\%) = \frac{\phi_0 - \phi_1}{\phi_0} \times 100 \tag{30}$$

The sign criteria are:

- Decrease in diameter: $\varepsilon\varphi > 0$
- Increase in diameter: $\varepsilon\varphi < 0$

4 DIAMETRICAL DEFORMATION ANALYSIS

The use of the Diametric Deformation is advantageous over simple analysis of the movements sustained by three or more lines of convergence. Among these advantages can be highlighted:

- The calculation is simple and does not require the installation of special geotechnical devices.
- It integrates the movements sustained by different points of the perimeter of the tunnel in a single value.
- The value obtained, it is expressed as percentage of deformation and is independent of the size of the excavation.
- It allows to directly compare the actual trend of the tunnel with results obtained in the stress-strain analysis.
- It allows to apply stability criteria, based on the percentage of rock mass deformation.

– In the case of deep tunnels, allows having an estimation of the load state of the support.

As a result of the observations and measurements by Sakurai (1983), it is usual to adopt the value of 1% as the deformation limit of the tunnel from which are observed signs of instability and difficulty in providing adequate support. However, as pointed out by Hoek (2001), this 1% limit shall be taken as an indication of increased difficulty in the excavation of tunnel and not as a strict limit that should not be exceeded. Ideally, the Diametric Deformation limit should be a parameter obtained from geomechanical calculations. Once this parameter and the conditions of application are established, it is extremely easy to design a monitoring plan based on alert levels.

Relative to the load level at which the support is subjected in a deep tunnel, Hoek (2001) reported the results of the influence of internal pressure support, p_i, in the deformation of the tunnel. The analysis was performed using axial-symmetric models in finite elements for a wide range of different types of rock mass, pressure fields and supports pressures. It is important to note that these analyzes are valid for tunnels at medium or high depths, not shallow tunnels.

Using curve-fitting techniques to the results of the analysis, it was obtained an approximate relation between the radial deformation of the tunnel (ε_t) and the ratio between pressure support (p_i) and the field pressure (p_0)

$$\varepsilon_t(\%) = 0.15\left(1-\frac{p_i}{p_0}\right)\left[\frac{\sigma_{cm}}{p_0}\right]^{-\left(3 p_i/p_0+1\right)/\left(3.8 p_i/p_0+0.54\right)} \quad (31)$$

The compressive strength of the rock mass (σ_{cm}) is a particularly appropriate parameter to evaluate the potential risk of instability that may experience a deep tunnel. As proposed by Hoek (2001), it can be calculated after the intact rock strength (σ_{ci}), the constant m_i and the Geological Strength Index (GSI). Hoek & Marinos (2000), described in detail how to obtain these parameters:

$$\sigma_{cm} = 0.0034\, m_i^{0.8}\, \sigma_{ci}\left[1.029 + 0.025\, e^{(-0.1 m_i)}\right]^{GSI} \quad (32)$$

In Figure 4, there is shown the variation of the radial deformation of the tunnel expressed in Equation 31, for different values of the ratio between the internal pressure of support (p_i) and the field pressure (p_0). It is understood that the rock mass exerts no load on the support when this ratio is zero ($p_i = 0$)

5 APPLICATION CASE

As a case of application of the methodology proposed, the excavation of the Tunnel of Larraskitu (Bilbao, Spain) is presented. It is a highway tunnel, 12 m wide, excavated in sedimentary rock, calcareous siltstones and calcareous sandstones. In the middle section of

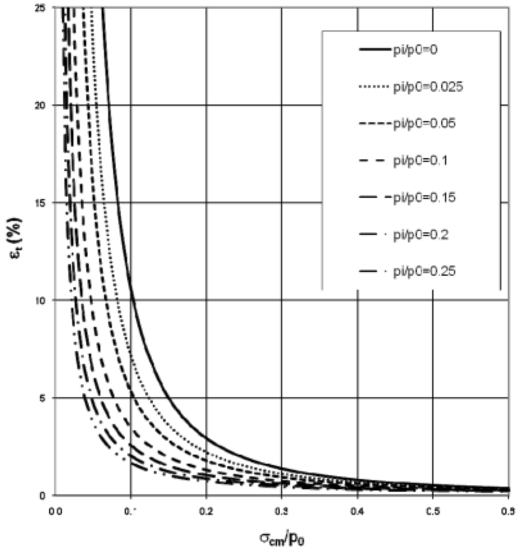

Figure 4. Influence of Internal Pressure of Support (p_i) in the deformation of the tunnel, after Hoek (2001).

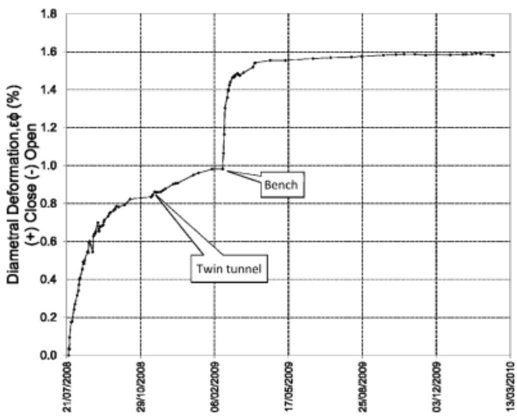

Figure 5. Benchmark section (CV-17) Tunnel of Larraskitu (Bilbao's South Metropolitan By-pass).

the tunnel, at depths around 200 m, large deformations were observed. Figure 5 presents the evolution of the Diametric Deformation of the tunnel in one of the benchmarks sections.

The graph shows the deformation caused by the excavation of the parallel tunnel, separated two diameters. At the moment when the deformation reached a 0.8% level, were evident the signals of support overload: load of the metallic arches, shotcrete cracks and plates bolts lost, being necessary the support reinforcement.

During the execution of tunnels, monitoring included geomechanical rock mass classification through Q and RMR indices, to determine the supports. Additionally GSI index was obtained. In addition to this geomechanical monitoring, there were

Table 1. CV-17 section parameters.

Overburden (m)	199
Q	0.02
GSI	25
m_i	8.3
γ (N/m^3)	27,300
σ_{ci} (MPa)	30

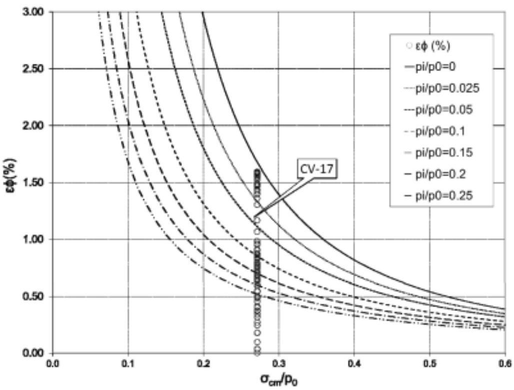

Figure 6. Evolution of Diametric Deformation in CV-17.

large number of laboratory test data, from the geotechnical characterization performed during the drafting. In the case of CV-17 section, the parameters used are shown in Table 1.

Applying Equation 32, the compressive strength of the rock mass is:

$$\sigma_{cm} = 1.47 \text{ MPa} \tag{33}$$

Assuming that the pressure field is equal to the lithostatic:

$$p_0 = 199 \times 27,300 = 5.43 \text{ MPa} \tag{34}$$

it is obtained:

$$\frac{\sigma_{cm}}{p_0} = 0.27 \tag{35}$$

In Figure 6 the Diametrics Deformations readings have been overlapped on the relationship proposed by Hoek (2001), see Equation 31.

About 0.8% Diametric Deformation, the ratio between the pressure of the support and the pressure field is around 0.1, that is the support provides a 10% of the lithostatic pressure. At the end of the excavation of the tunnel, when the tunnel was stabilized and before installing the final lining, this ratio was very close to zero. That is, the support was not exercising load to have reached equilibrium with the rock mass.

6 CONCLUSIONS

In this paper, we have obtained the analytical expression of the Diametric Deformation (ε_ϕ) in a tunnel from the measurements obtained with benchmarks, see Equation 29 and 30.

From this value, we propose a methodology for stress-strain control of the tunnel, based on the estimation of a diametric deformation limit value that can assume the support. As a general rule, we propose the 1% limit proposed by Sakurai, as indicative of the existence of instabilities and possible support problems. However, this value must be determined in each case by stress-strain analysis, preferably by numerical methods.

Also the use of the expression given by Hoek (2001) which correlates the support Internal Pressure (p_i) with the deformation of the tunnel has been analyzed. In the example presented, this expression correlates quite well with that observed during the excavation of the tunnel.

It is for further study on the proposed methodology for cases tunneling in highly anisotropic rock mass or a ratio between horizontal and vertical pressures other than one.

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